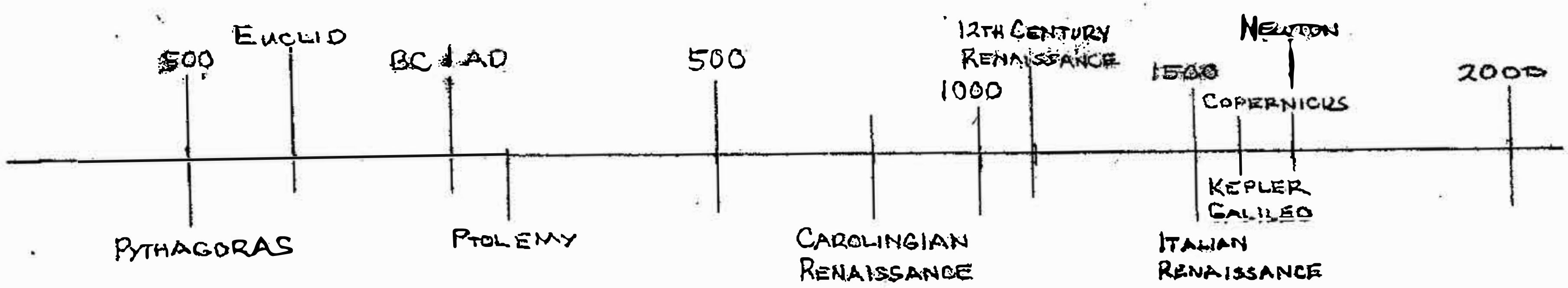
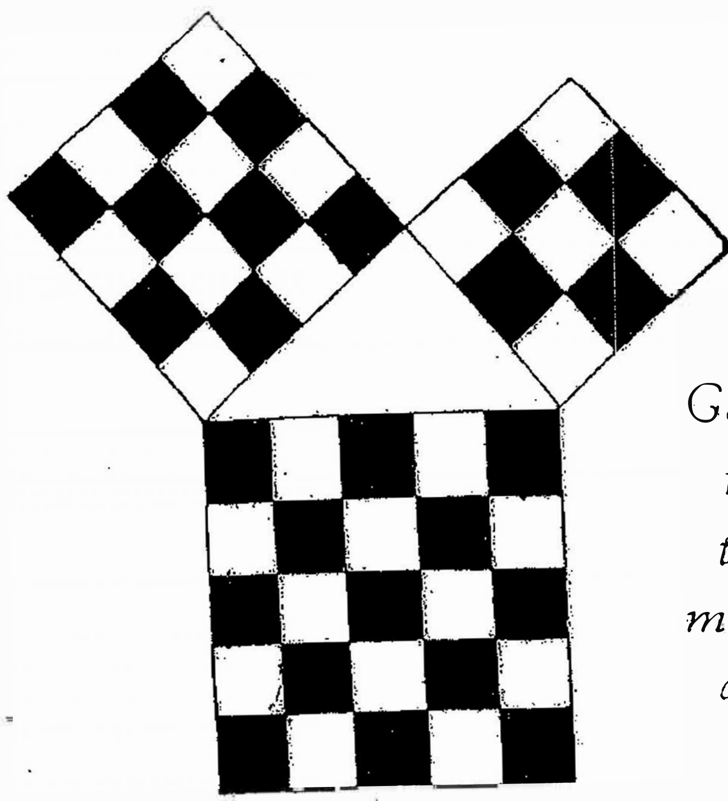
Geometry

Geometry has been called the first science. It is as old as humans. Geometry...geo is earth...metry is measure. Earth measure is surveying. Surveying is a requirement for civilization to occur in a river valley. Civilization requires taxation to fund a government. Rivers often flood the fertile valley. Boundaries must be quickly and accurately reestablished to prevent disruption in the revenue to the government.

Another use of geometry in every river valley civilization is to build structures to honor a deity or deities. Temple building geometry was often accomplished with three simple tools...a compass, a straightedge and a marking device. All dimensions of a temple were derived by geometrically harmonizing from a declared radius of a circle whose length became the side of a unit square or square one. When we say "Let's go back to square one" we rarely mention that we are referring to an ancient geometric process of harmony and proportion in designing temples.

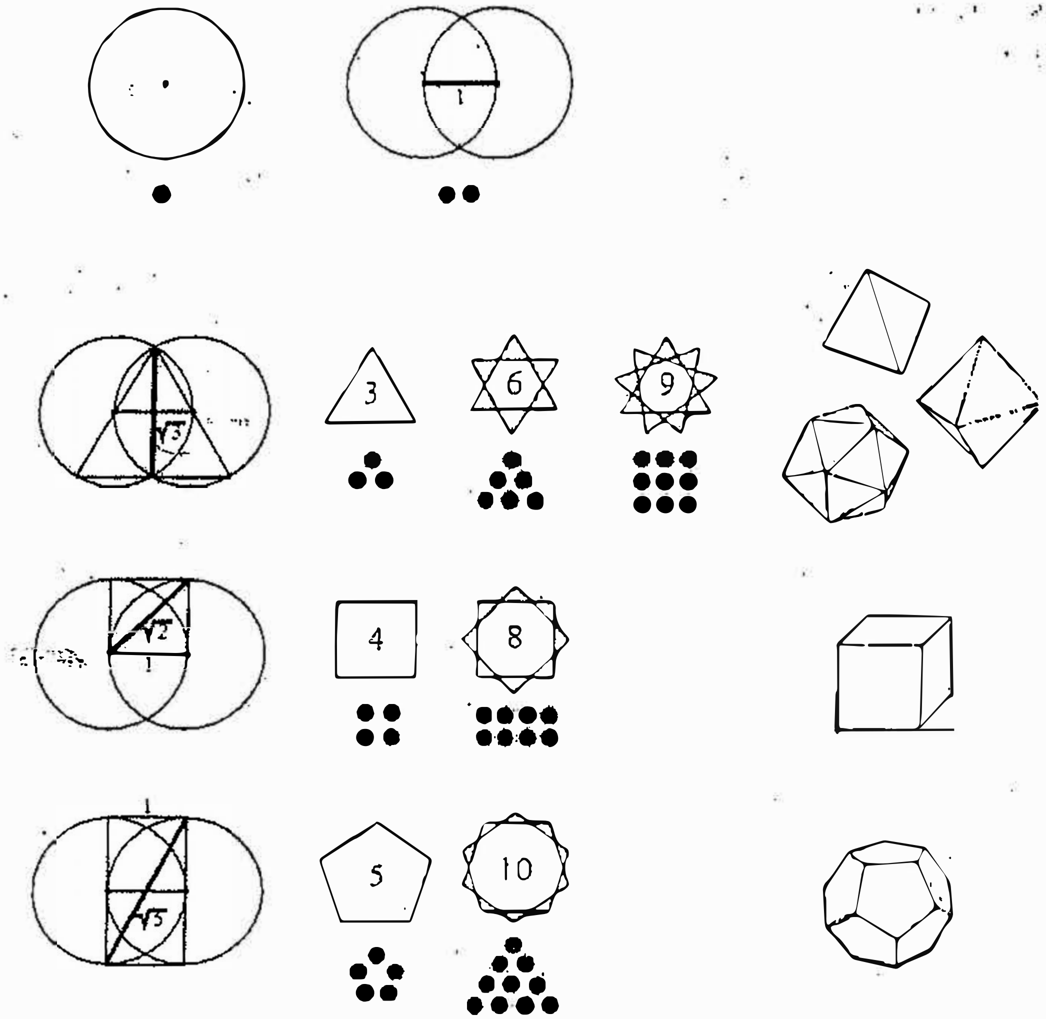


THE FORTY-SEVENTH PROBLEM OF EUCLID



Geometry has two great treasures: one is the theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel.

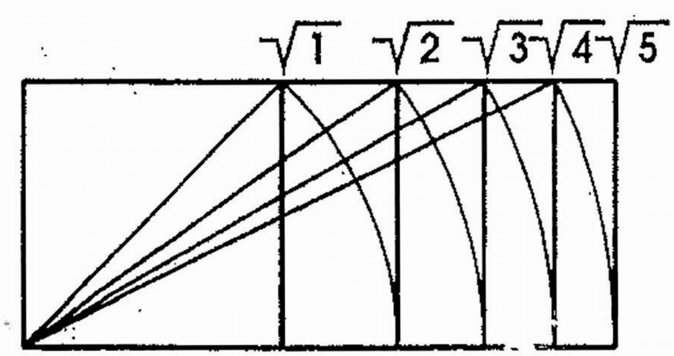
JOHANNES KEPLER



Two intersecting circles contain all the proportions necessary to generate the polygons found in nature and the five Platonic volumes.

Calculate Square Roots

You can use the Pythagorean Theorem to prove that each new rectangle is a Square Root Rectangle. That is, their long and short sides form the ratios (beginning with the Square), Root 1, Root 2, Root 3, Root 4, Root 5 and so on. Each of these "roots" have numerical values. Here are approximations, but use your calculator to find more decimal places for each, and extend their values below.



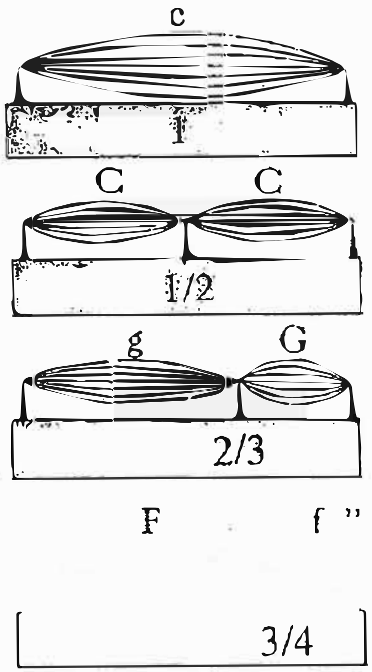
$\sqrt{5} = 2.236$

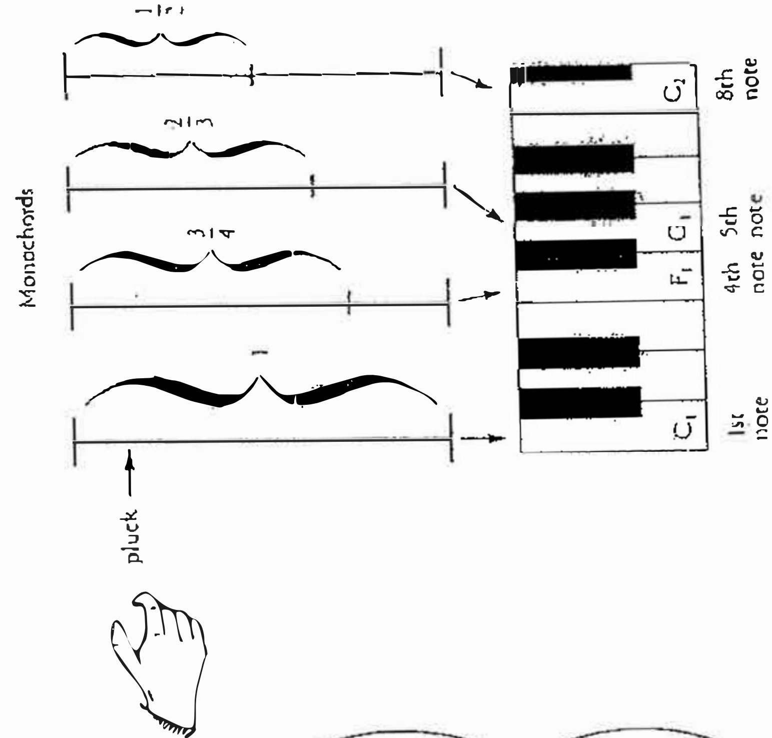
$\sqrt{4} = 2.000$ = Double Square

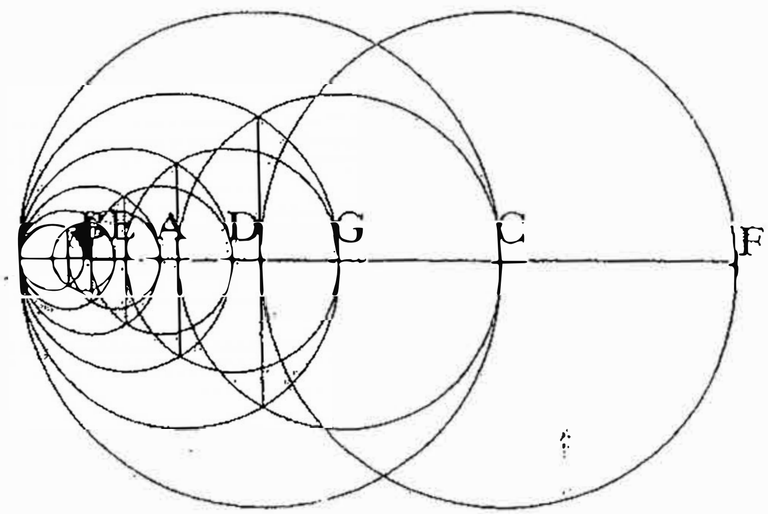
$\sqrt{3} = 1.732$

$\sqrt{2} = 1.414$

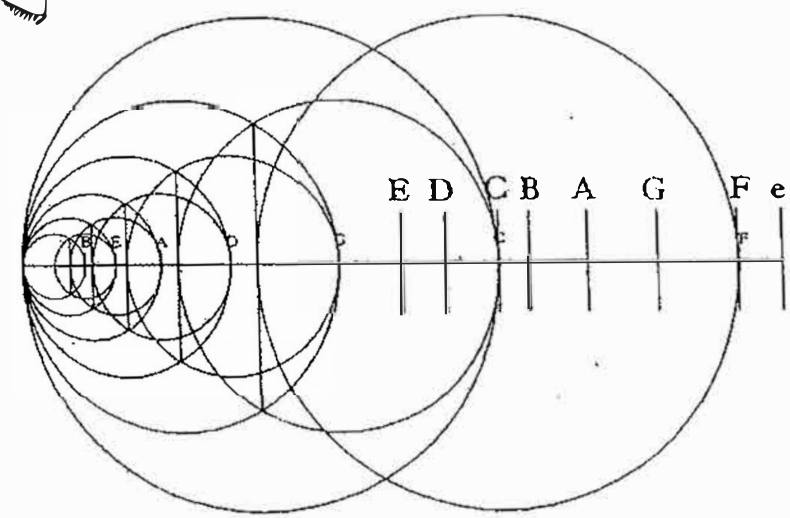
$\sqrt{1} = 1.000$ = Square







These seven notes span three octaves



Doubling the lengths of shorter strings retains the note but lowers its octave, bringing all notes within the same circle and octave

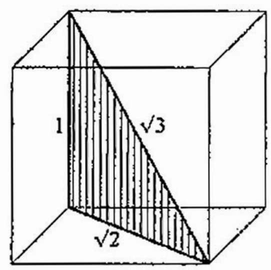


Fig. 14

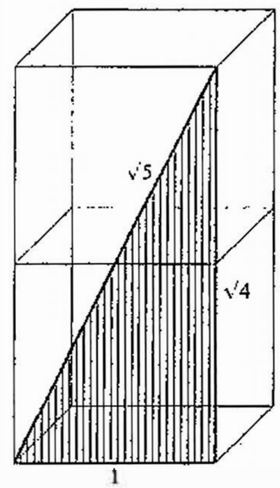


Fig. 15

- Locate a diagonal of a double square face.
- Locate a diagonal through the body of the double cube.

If the short edge of the double cube equals 1, the diagonal through a double square face equals √5, and the diagonal through the body of the double cube equals √6 (fig. 16).[7]

* * *

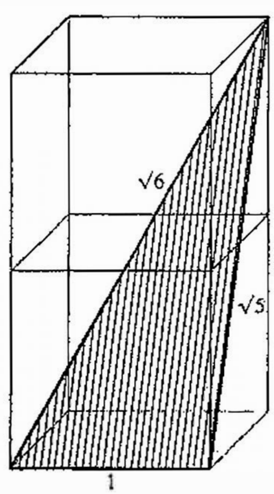


Fig. 16

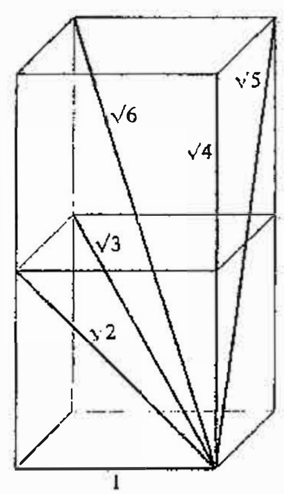
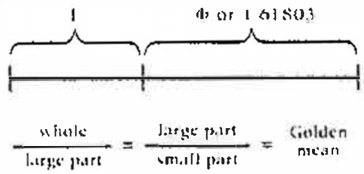
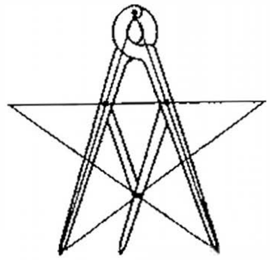
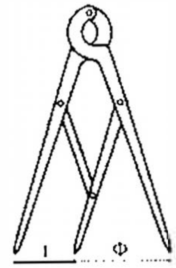


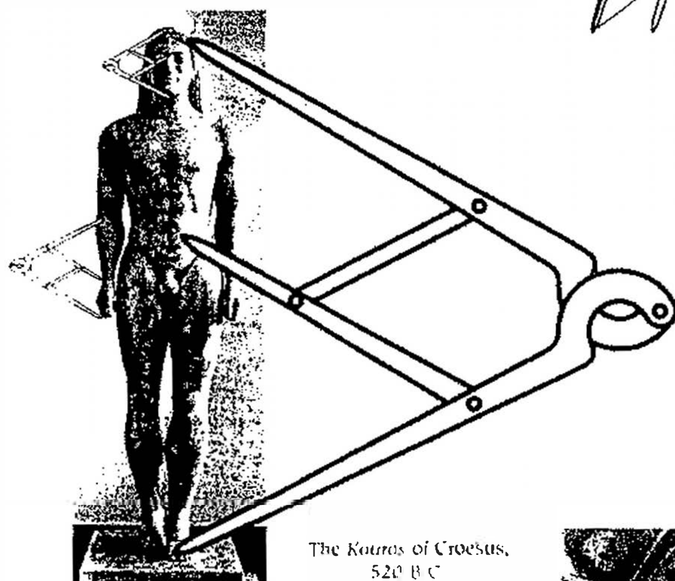
Fig. 17

Introduction to the Golden Mean

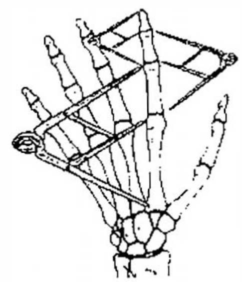




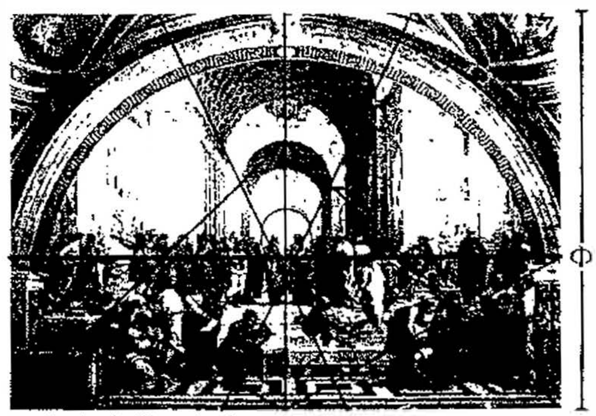




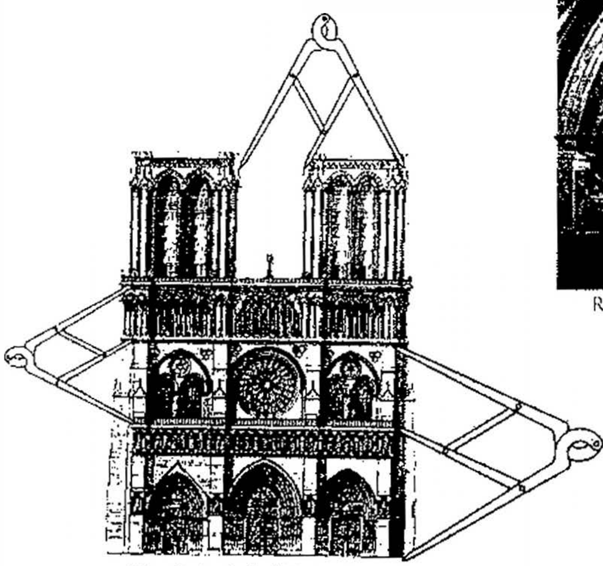
The *Kouros* of Croesus, 520 B.C



Finger bones exhibit Φ



Raphael's *The School at Athens*, harmonic analysis



The Cathedral of Notre Dame.

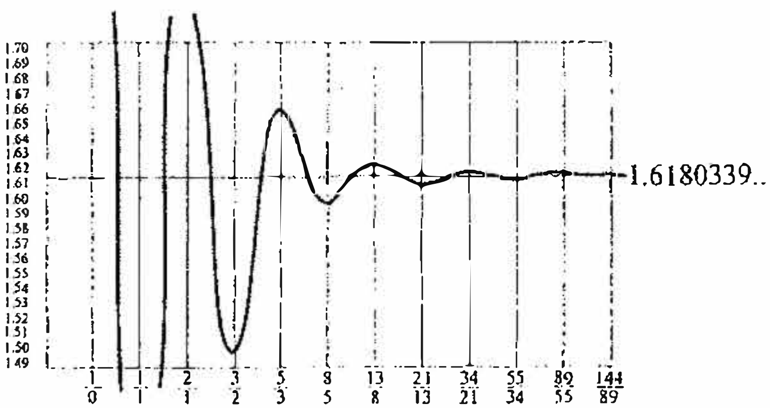
THE GOLDEN MEAN

Write out the first few terms of the Fibonacci sequence. Draw a line under each to make it a fraction, and underneath write the Fibonacci sequence shifted back one term.

$\frac{1}{0} \quad \frac{1}{1} \quad \frac{2}{1} \quad \frac{3}{2} \quad \frac{5}{3} \quad \frac{8}{5} \quad \frac{13}{8} \quad \frac{21}{13} \quad \frac{34}{21} \quad \frac{55}{34}$

Use a calculator to divide and convert each fraction into decimal form.

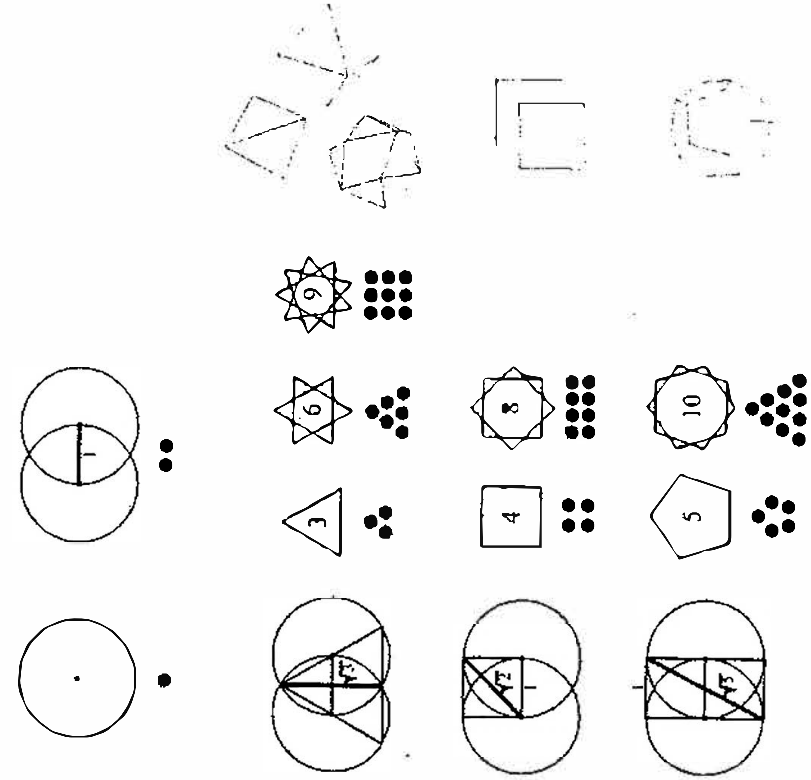
A graph of the results shows each term getting closer to an ideal of 1.61803398875 . . . or rounded off to 1.618 or even 1.62. Notice how it begins crudely, passing far over then



Historical names of the Golden Mean:

Plato	*the section*
Euclid	*the extreme and mean ratio*
Romans	*aurea sectio (golden section)*
Luca Pacioli	*the divine proportion*
Christopher Clavius	*Godlike proportion*
Johannes Kepler	*the divine section*
Johann F. Lorentz	*the continued division*
J. Leslie	*the medial section*
Adolf Zeising	*the golden cut*
Mark Barr	Φ *(phi)*

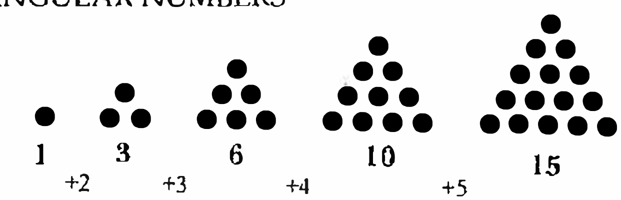
Avoid extremes—keep the Golden Mean.
—*Cleobulus of Lindus (sixth century* B.C., *one of the Seven Wise Men of ancient Greece)*



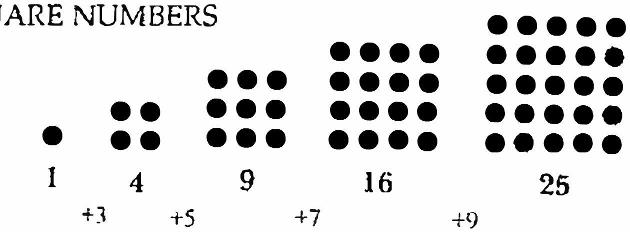
Two intersecting circles contain all the proportions necessary to generate the polygons found in nature and the five Platonic volumes.

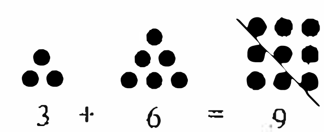
Figurate Arithmetic. Simple arrangements of pebbles showed ancient mathematicians the patterns inherent in relationships among numbers. For example, any two consecutive "triangular" numbers (1, 3, 6, 10, 15 . . .) added together always make a "square" number (1, 4, 9, 16, 25 . . .).

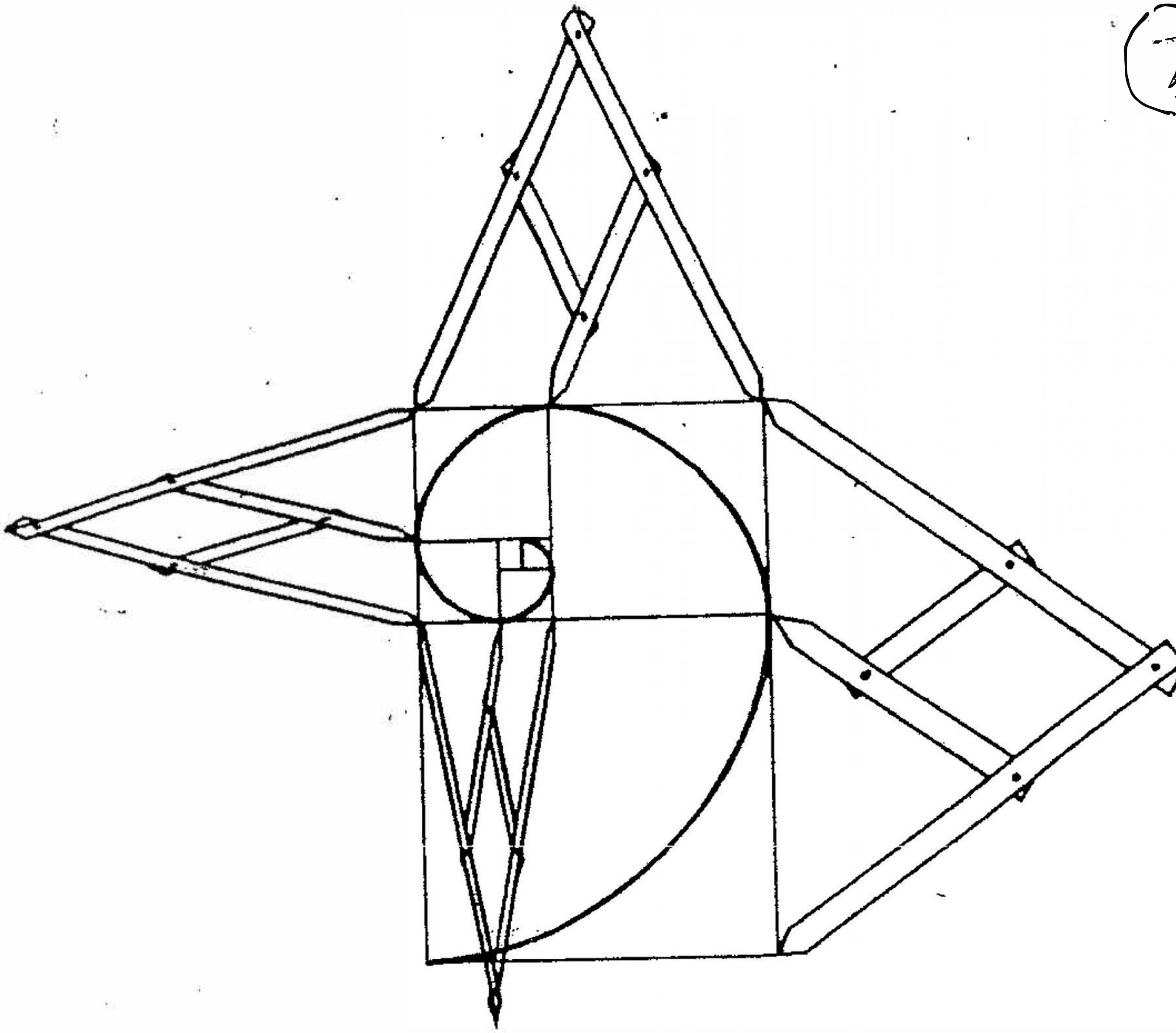
TRIANGULAR NUMBERS

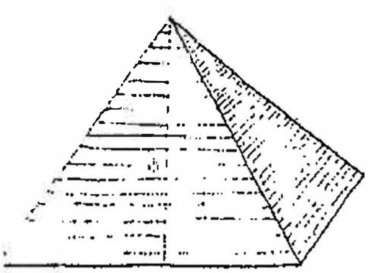


SQUARE NUMBERS

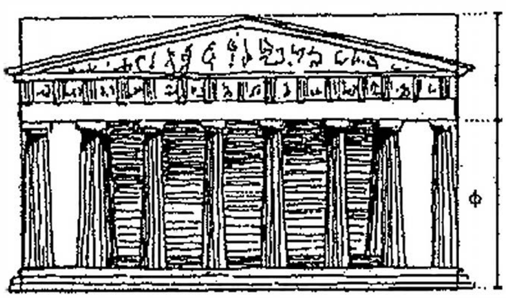




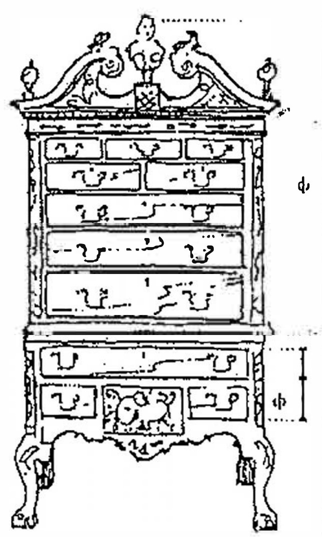




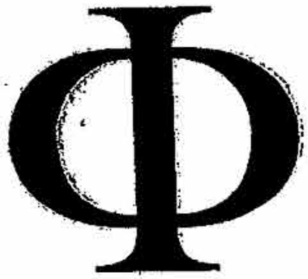
PHI IN THE PYRAMIDS
The Great Pyramid of Giza is constructed with the golden ratio at its core. The height of its side is equal to 1.618 times the length of half its base.



PHI IN THE PARTHENON
The Parthenon uses the golden ratio for its overall dimensions. When squared, it leaves a second, smaller golden rectangle, which when squared determines the height of the columns. Many other elements and details were determined with this method.



PHI IN THE HIGHBOY
The high chest of drawers, known as the Pompadour, made in Philadelphia between 1762 and 1790, uses the golden ratio to determine many of its measurements. The carcase is a golden rectangle. The position of the waist is determined by dividing the overall height by phi. And the two lower drawers also are golden rectangles.



The Greek symbol Φ is the equivalent of the English sound "ph". It sounds like fly without the "l" and is used to represent the proportion that Euclid described in Chapter 5 of *Elements*. This proportion is called the Divine Proportion. It is also known as the Golden Proportion, the Golden Mean and the Golden Ratio

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Adolf Zeising	*the golden cut*
Mark Barr	Φ *(phi)*

The Fibonacci sequence has fascinating number properties. For example, its part resembles the whole. Use a calculator to divide the twelfth term, 89, into unity. The result is an endless decimal that, broken into parts, replicates the entire Fibonacci sequence: (1/89 = .01123581321345589144 . . .)

Although this proportion appears to have been understood for centuries, it was first articulated mathematically by Euclid of Alexandria (ca. 325–265 B.C.E.) in his book, *Elements*. In the fifth chapter, Euclid drew a line and divided it into what he called its "extreme and mean ratio":

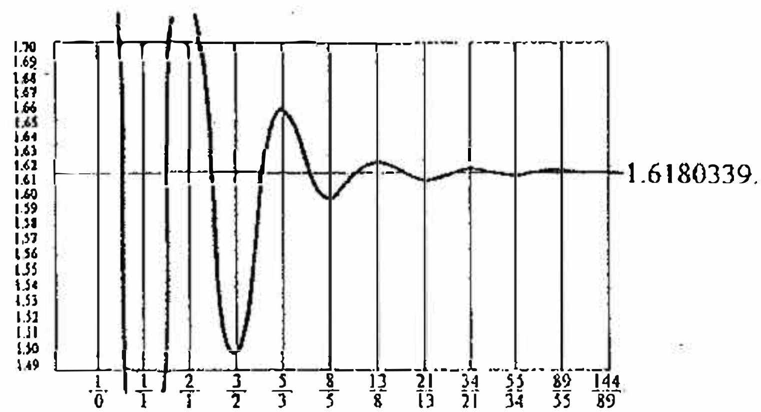
> *A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segments, so is the greater to the lesser.*

In other words AB / AC = AC / CB. When we speak of this equation we say that AB is to AC as AC is to CB.

This proportion, articulated by Euclid, tells us that the ratio of the whole line AB to its larger part AC is the same as the ratio of the larger part AC to the smaller part CB. When calculated mathematically this gives a ratio of approximately 1.61803 to 1, which can also be expressed as $\frac{1+\sqrt{5}}{2}$ or Φ.

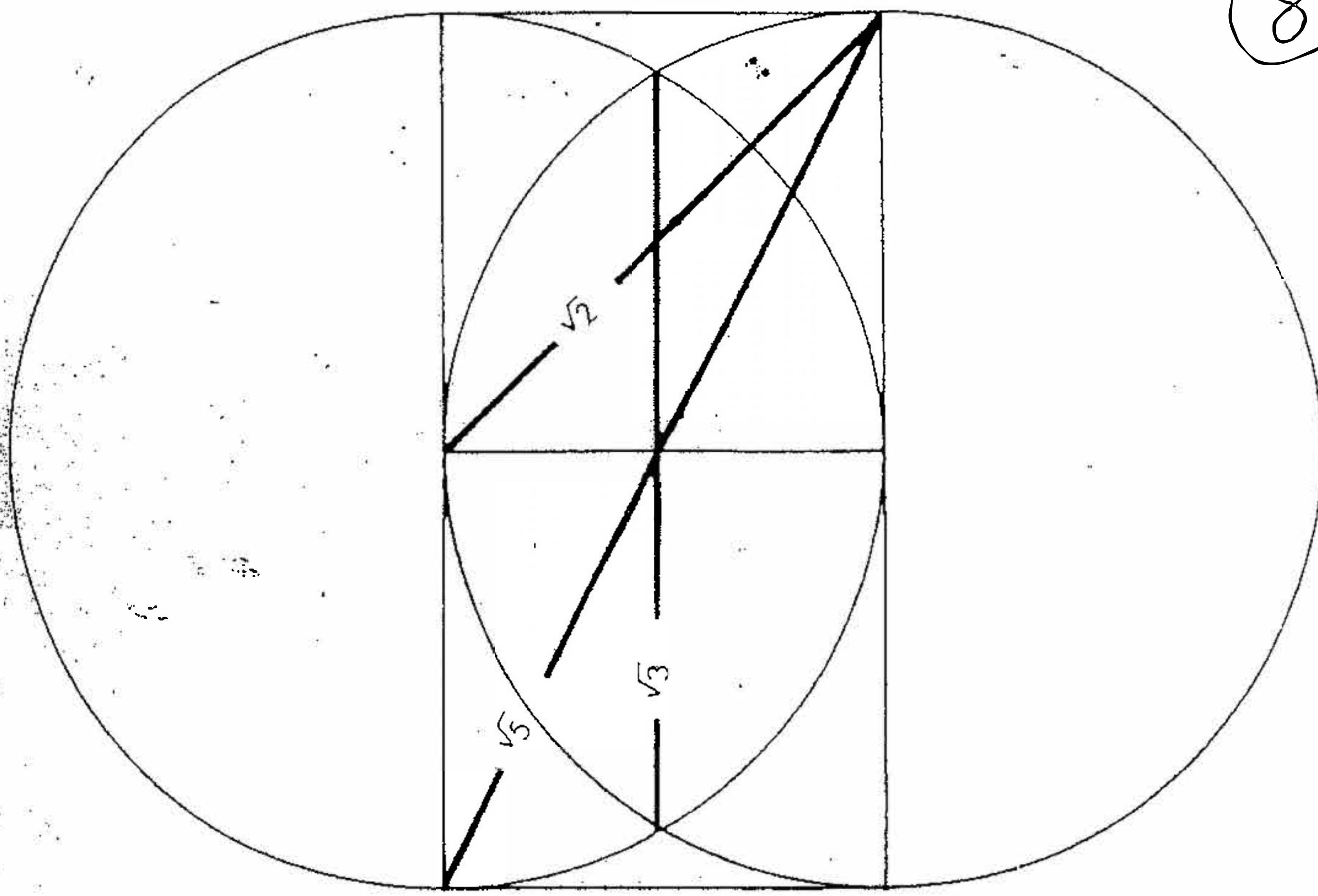
Fibonacci sequence:

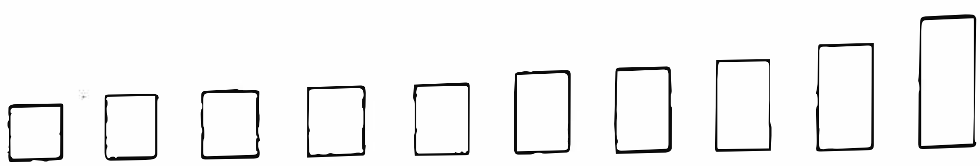
0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597

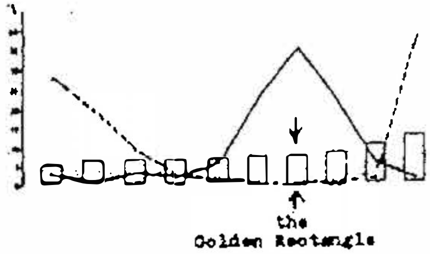


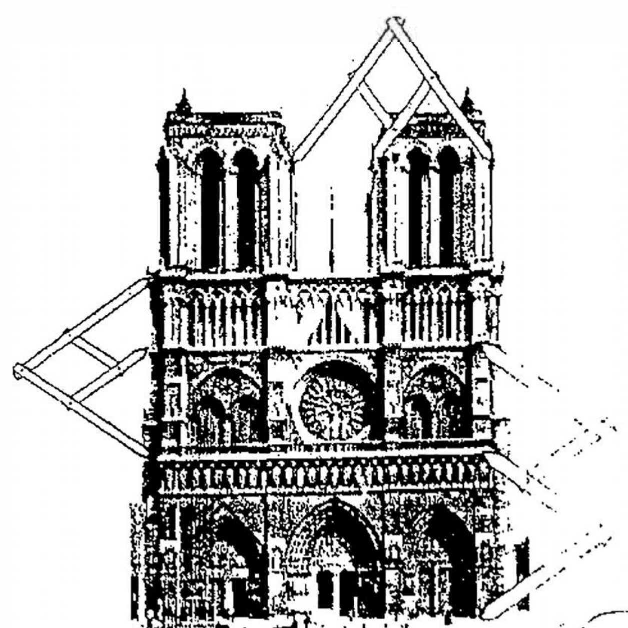
Avoid extremes---keep the Golden Mean.
Cleobulus of Lindus (sixth century B.C., one of the Seven Wise Men of ancient Greece)

"There's a limit, a measure in things," which the intelligent man will neither fall short of nor exceed.
Horace (65-8 B.C., Satires, I, 1.105)


$\sqrt{2}$
$\sqrt{3}$
$\sqrt{5}$




the
Golden Rectangle



To develop a complete mind:
Study the science of art;
Study the art of science.
Learn how to see.
Realize that everything
connects to everything else.

- leonardo da vinci